Playing General Structure Rewriting Games

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AGI
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Motivation

A system is intelligent if it is more useful to talk about it in terms of goals than in terms of mechanisms.

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Mentioned Models of Computation (deterministic)

Turing Machines

- AIXI

Term Rewriting

- OpenCog

Graph Rewriting

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- Non-determinism with **multiple players** (and probability)
- Hypergraph **rewriting** with constraints
- Continuous dynamics (by **ODE**) in the model

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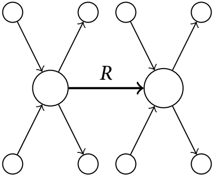
Why a high-level model? Easier to program, debug, understand

Cf. programming languages, regular expressions, databases

Structure Rewriting Rules

Rewriting Example



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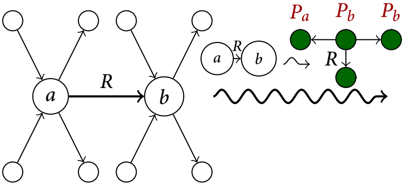

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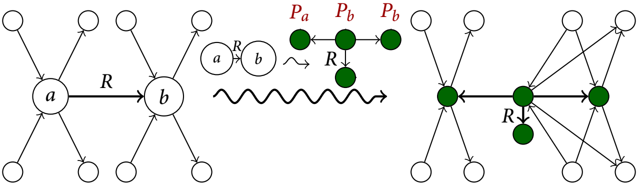

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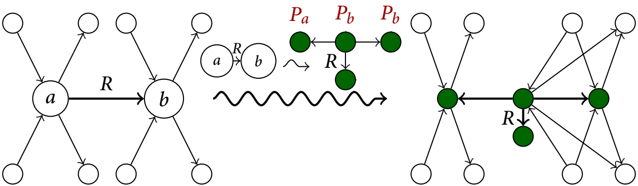

Embedding: σ is **injective** and $R_i^{\mathfrak{A}}(a_1, \dots, a_{r_i}) \Leftrightarrow R_i^{\mathfrak{B}}(\sigma(a_1), \dots, \sigma(a_{r_i}))$

$$\sigma \ : \quad \mathfrak{A} = (A, R_1^{\mathfrak{A}}, R_2^{\mathfrak{A}}, \dots, R_k^{\mathfrak{A}}) \quad \hookrightarrow \quad (B, R_1^{\mathfrak{B}}, R_2^{\mathfrak{B}}, \dots, R_k^{\mathfrak{B}}) = \mathfrak{B}$$

Rewriting: $\mathfrak{B} = \mathfrak{A}[\mathfrak{L} \to \mathfrak{R}/\sigma]$ iff $B = (A \setminus \sigma(L)) \dot\cup R$ and,

for $M = \{(r, a) \mid a = \sigma(l), r \in P_l^{\mathfrak{R}}$ for some $l \in L\} \cup \{(a, a) \mid a \in A\}$,

$(b_1, \dots, b_{r_i}) \in R_i^{\mathfrak{B}} \ \Leftrightarrow \ (b_1, \dots, b_{r_i}) \in R_i^{\mathfrak{R}}$ **or** $(b_1 M \times \dots \times b_{r_i} M) \cap R_i^{\mathfrak{A}} \neq \varnothing$.

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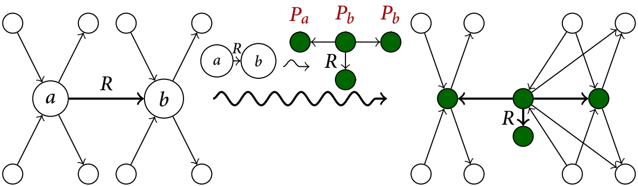

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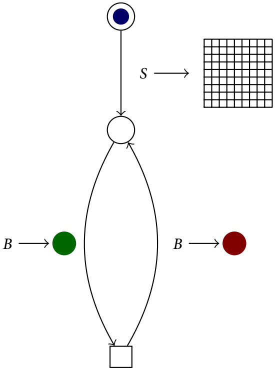
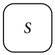
Motivation: many questions **naturally defined** as **structure rewriting games**:
constraint satisfaction, model checking, graph algorithms, games people play

Example System: Gomoku (Connect–5)





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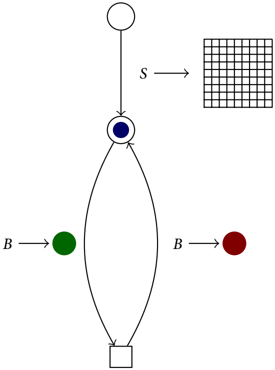

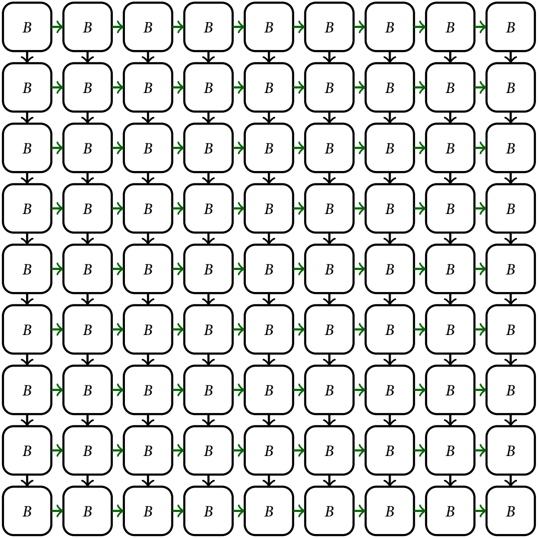

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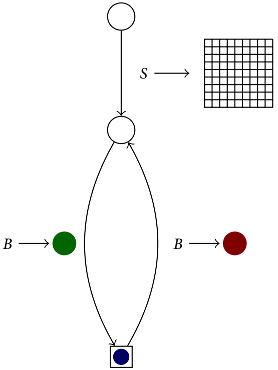

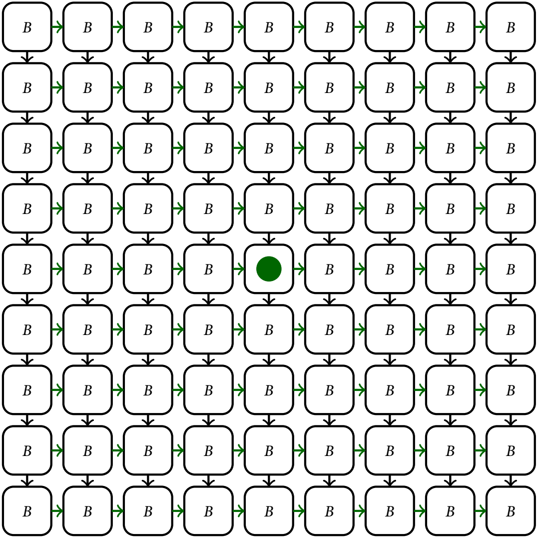

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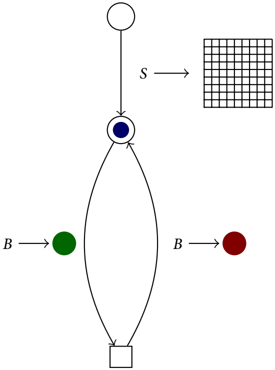

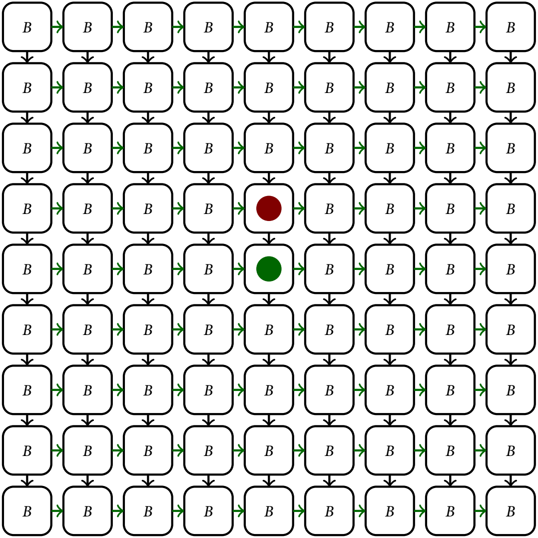

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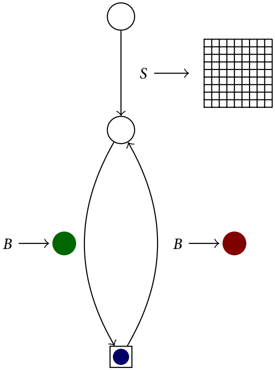

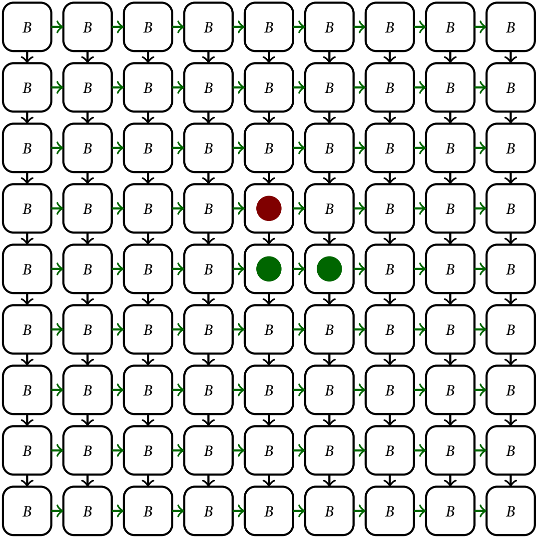

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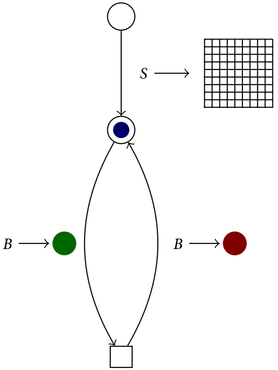

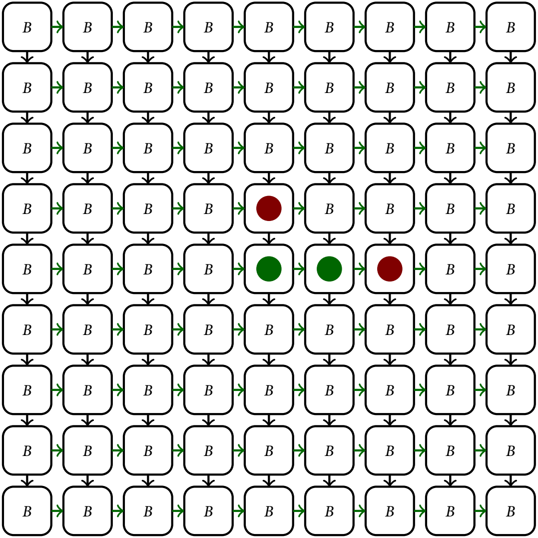

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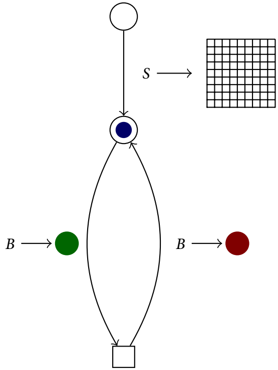

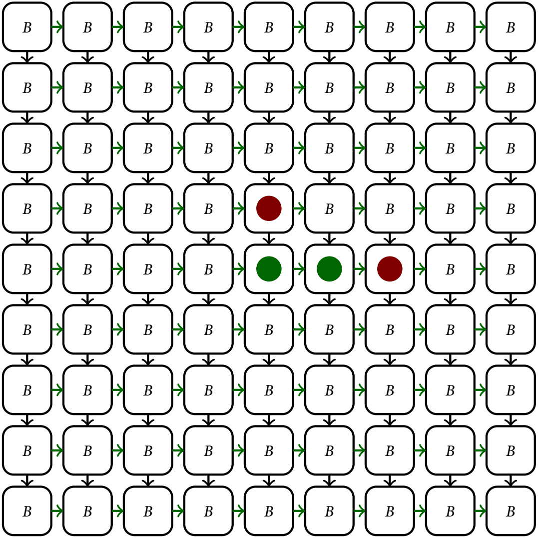

$$\exists x_1 \dots x_5 \Big(\bigwedge_{1\le i\le 5} G(x_i) \wedge \Big(\bigwedge_{1\le i\le 5} R(x_i, x_{i+1}) \vee \bigwedge_{1\le i\le 5} C(x_i, x_{i+1}) \vee$$

$$\bigwedge_{1\le i\le 5} \exists y (R(x_i, y) \wedge C(y, x_{i+1})) \vee \bigwedge_{1\le i\le 5} \exists y (R(x_i, y) \wedge C(x_{i+1}, y))\Big)\Big)$$

Continuous Dynamics

$\mathbb{R}$-structures: $\mathfrak{A} = (A, R_1, \ldots, R_k, f_1, \ldots, f_l)$ with $f_i : A \to \mathbb{R}$

Additional Parameters to a Rule:

- **dynamics**: system of **ordinary differential equations**
- **updates**: equations assigning **values on the right-hand side**
- **constraints**: precondition, invariant, postcondition

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Logic

- **Monadic Second-Order Logic** (MSO):
$$\forall X(x \in X \wedge (\forall z, v(z \in X \wedge R(z,v) \to v \in X)) \to y \in X)$$
- **Real-valued terms with counting**: $2 \cdot \chi\big(\exists y(P(y) \wedge R(x,y))\big) + f(x)$
- **Real quantification**: $\exists a \in \mathbb{R}(a^2 \cdot f(x) + a - 1 = 0) \wedge (f(x) > 2)$

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Semantics of Application

(1) All dynamics applies **concurrently**

(2) Rules with **minimal time** fire

(3) Discrete rewriting **after** continuous evolution

Running the System

How do the players play? Monte Carlo with UCT and Learned Playout

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- **Both players** play **randomly** a large number of times
- Calculate the **ratio of wins** of each player

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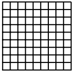

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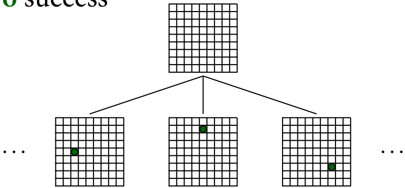

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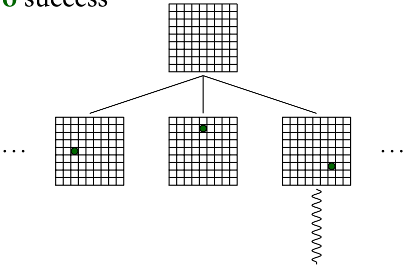

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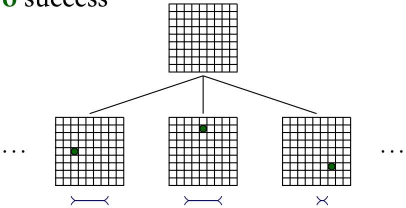
Pick Max Upper Confidence

$$C \cdot \sqrt{\frac{\ln(n(v)+1)}{n(w)+1}}$$



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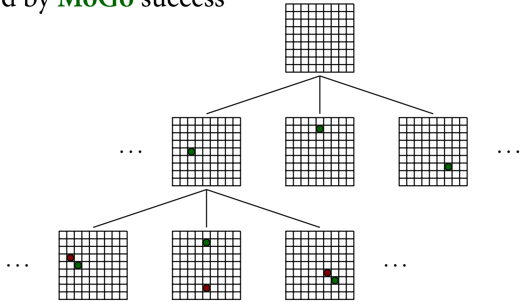

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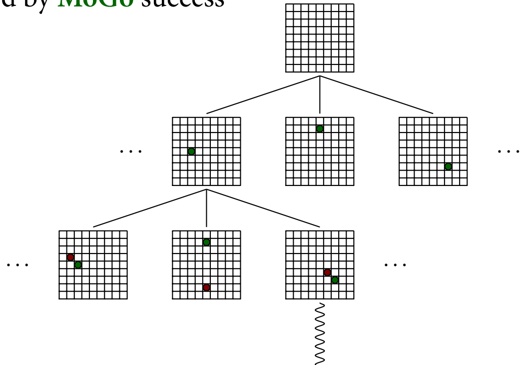

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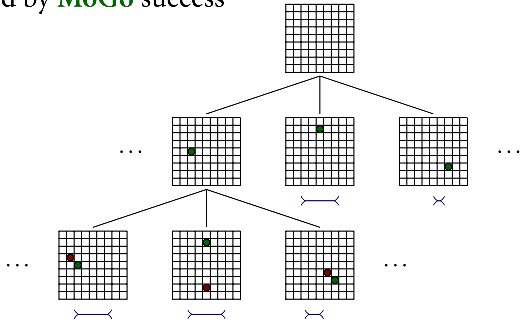

UCT for Structure Rewriting Games

Improving Playout Policy

- **Hints: formulas** which separate **good and bad** states (**moves**)
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- Formula should **separate** winning from loosing states
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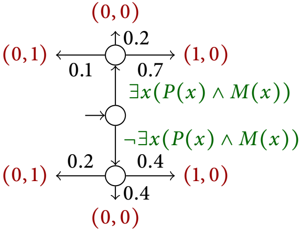
Evaluation Games

Improvements vs. UCT with random

- **Breakthrough**: **beat if possible**
  ca. 70% improvement
- **Gomoku**: **play near your stone**
  ca. 80% improvement



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