Playing Structure Rewriting Games with Formulas on States

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Ongoing work with help from D. Fischer, T. Ganzow, E. Abraham, U. Loup, Ł. Stafiniak

Mathematische Grundlagen der Informatik
RWTH Aachen

AlgoSyn

Aachen, 2009

Structure Rewriting Rules

Relational Structures and Embeddings

$$\sigma \ : \quad \mathfrak{A} = (A, R_1^{\mathfrak{A}}, R_2^{\mathfrak{A}}, \ldots, R_k^{\mathfrak{A}}) \quad \to \quad (B, R_1^{\mathfrak{B}}, R_2^{\mathfrak{B}}, \ldots, R_k^{\mathfrak{B}}) = \mathfrak{B}$$

Embedding: σ is **injective** and $R_i^{\mathfrak{A}}(a_1, \ldots, a_{r_i}) \Leftrightarrow R_i^{\mathfrak{B}}(\sigma(a_1), \ldots, \sigma(a_{r_i}))$

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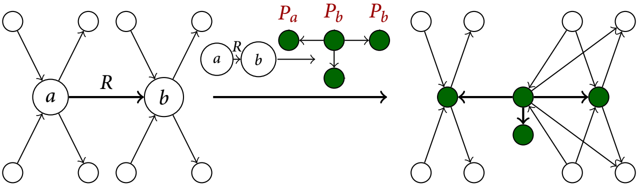
Rewriting Definition

$\mathfrak{B} = \mathfrak{A}[\mathfrak{L} \to \mathfrak{R}/\sigma]$ iff $B = (A \setminus \sigma(L)) \dot{\cup} R$ and,
for $M = \{(r, a) \mid a = \sigma(l), r \in P_l^{\mathfrak{R}} \text{ for some } l \in L\} \cup \{(a, a) \mid a \in A\}$,

$$(b_1, \dots, b_{r_i}) \in R_i^{\mathfrak{B}} \;\Leftrightarrow\; (b_1, \dots, b_{r_i}) \in R_i^{\mathfrak{R}} \textbf{ or } (b_1 M \times \dots \times b_{r_i} M) \cap R_i^{\mathfrak{A}} \neq \emptyset.$$

(in the second case at least one $b_j \notin \mathfrak{A}$)

Rewriting Example



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Game arena is a **directed graph** with:

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- edges **labelled by rewriting rules**

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- **Existential**: $\mathfrak{A}_{\text{next}} = \mathfrak{A}[\mathfrak{L} \to \mathfrak{R}/\sigma]$, the player chooses the embedding σ
- **Universal**: $\mathfrak{A}_{\text{next}} = \mathfrak{A}[\mathfrak{L} \to \mathfrak{R}]$, **all** occurrences of $\mathfrak{L}$ are rewritten to $\mathfrak{R}$

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Winning conditions:

- L_μ (or temporal) formula ψ with **MSO sentences** for predicates, or
- MSO formula φ to be evaluated on the **limit** of the play
  Limit of $\mathfrak{A}_0\mathfrak{A}_1\mathfrak{A}_2 \ldots = \left(\bigcup_{n\in\mathbb{N}} \bigcap_{i\geq n} A_i,\ \bigcup_{n\in\mathbb{N}} \bigcap_{i\geq n} R^{\mathfrak{A}_i}\right)$
- **Reach φ**: Player 1 **wins** if the play reaches $\mathfrak{A}$ s.t. $\mathfrak{A} \models \varphi$

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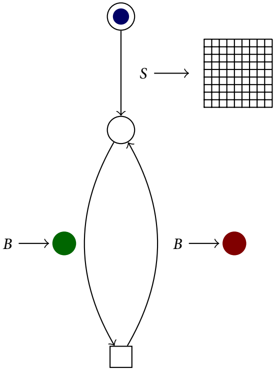
Motivation: many questions are **naturally defined as such games**:
constraint satisfaction, model checking, graph measures, games people play

Example Game: Gomoku (Connect–5)



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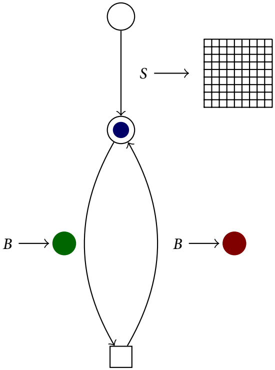

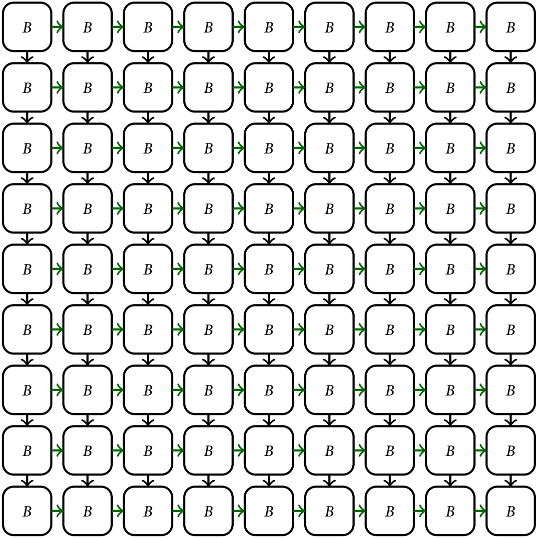

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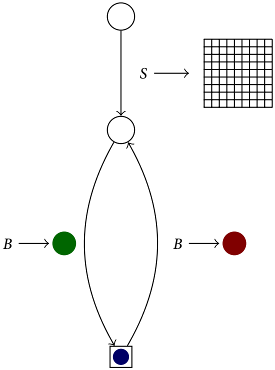

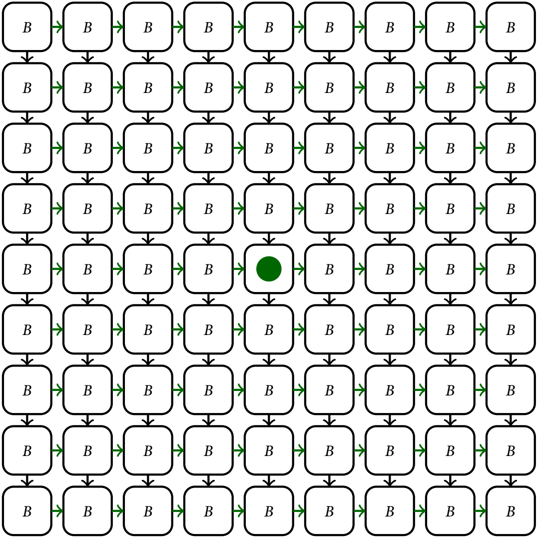

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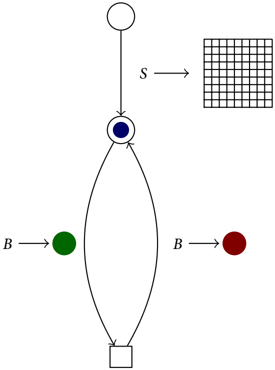

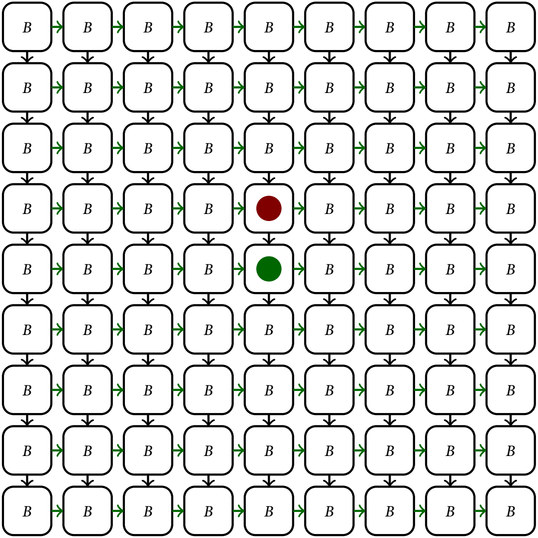

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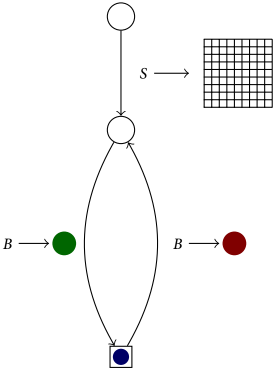

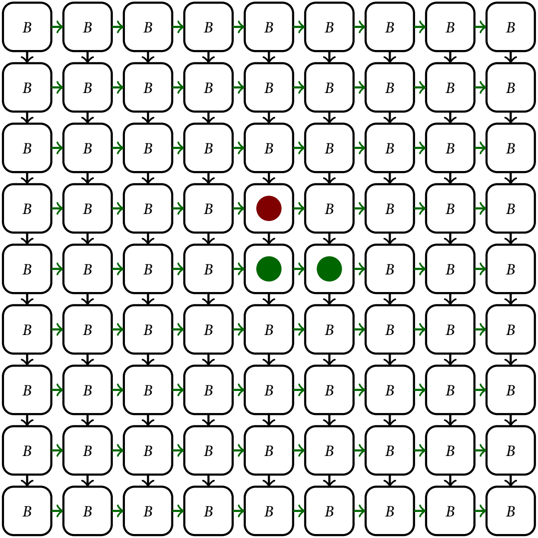

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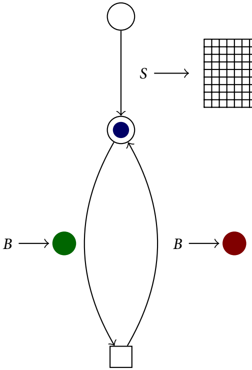

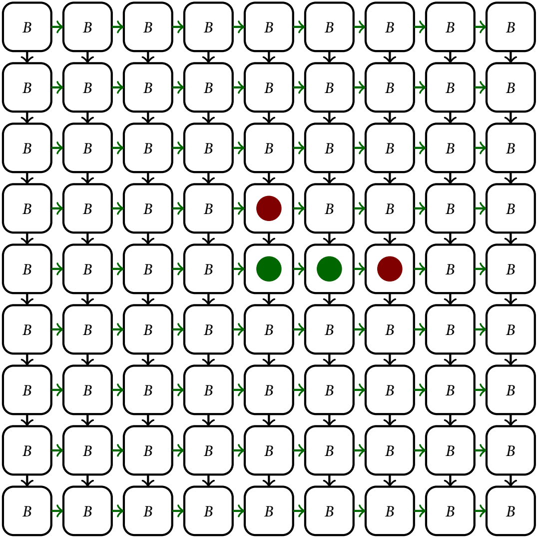

$$\exists x_1 \dots x_5 \Big(\bigwedge_{1 \le i \le 5} G(x_i) \wedge \Big(\bigwedge_{1 \le i \le 5} R(x_i, x_{i+1}) \vee \bigwedge_{1 \le i \le 5} C(x_i, x_{i+1}) \vee \bigwedge_{1 \le i \le 5} \exists y (R(x_i, y) \wedge C(y, x_{i+1})) \vee \bigwedge_{1 \le i \le 5} \exists y (R(x_i, y) \wedge C(x_{i+1}, y)) \Big) \Big)$$

Simple Structure Rewriting

Separated Structures: no element is in two non-terminal relations **(Courcelle, Engelfriet, Rozenberg, 1991)**

Separated: $\circ \xrightarrow{R} \circ \xrightarrow{a} \circ \xrightarrow{R} \circ$

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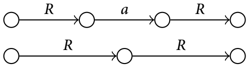

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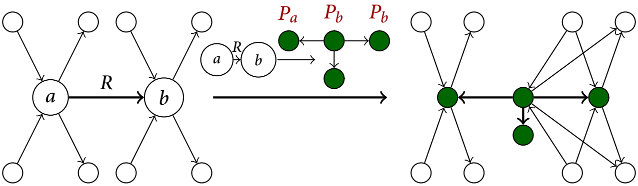

Previous Result

Logics

- $L_\mu[\mathrm{MSO}]$**:** Temporal properties expressed in L_μ (subsumes LTL) with properties of structures (states) expressed in MSO
- $\lim \mathrm{MSO}$**:** Property of the limit structure expressed in MSO

Theorem

- *Let R be a **finite** set of **(universal) simple structure rewriting rules,***
- *and φ be an $L_\mu[\mathrm{MSO}]$ or $\lim \mathrm{MSO}$ formula.*

*Then the set $\{\pi \in R^\omega : (\lim)S(\pi) \models \varphi\}$ is **ω-regular**.*

Corollary

Establishing the winner of (universal) finite simple rewriting games is decidable.

Implementing the Result

Using Tree Automata

- **Tool**: **MONA**
  - Developed at **BRICS** since **1996** by Nils Klarlund and Anders Møller
  - Symbolic representation with **BDDs**
  - **Minimisation** at each step
- **Example:** a simple **tic-tac-toe** game
- **Result:** **memory overflow** on 2×2 grid
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Present Approach

- Use **simulation** to detect **promising moves**
- Construct a **good (not necessarily optimal)** strategy
- **Perspective:** prove that the strategy is **winning**

Simulation-Based Game Playing

Game Playing Methods

- **General pattern**: **Minimax**
- ***α*-*β* pruning** and other optimisations
- Multiple special **heuristics**, **opening tables**
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How to determine the **value of a position** v in a **general** game?

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Immediate Problems:

- Makes **trivially stupid** moves
- Very **flat lookahead**

Upper Confidence Bounds for Trees

Build a tree during Monte-Carlo

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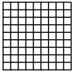

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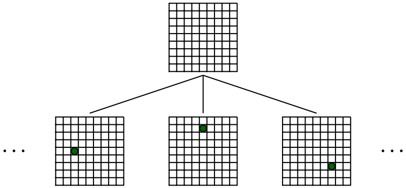

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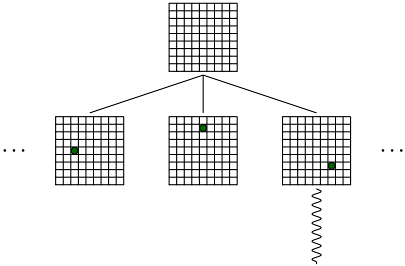

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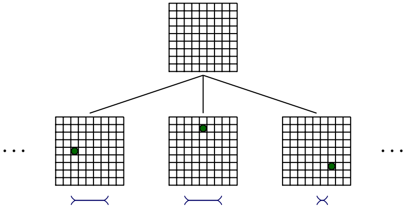
Pick Max Upper Confidence

$$C \cdot \Delta \cdot \sqrt{\frac{\ln(n(v)+1)}{n(w)+1}}$$



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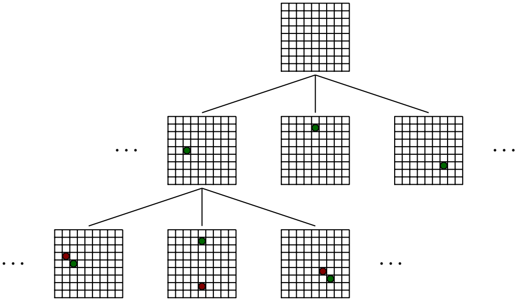

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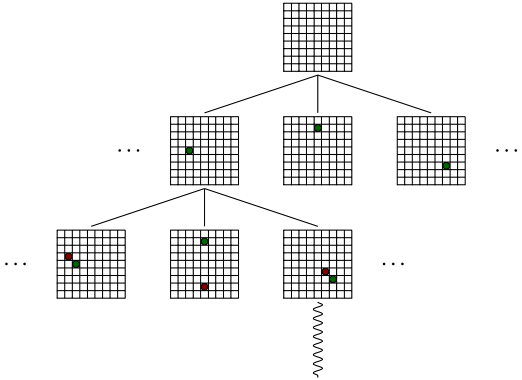

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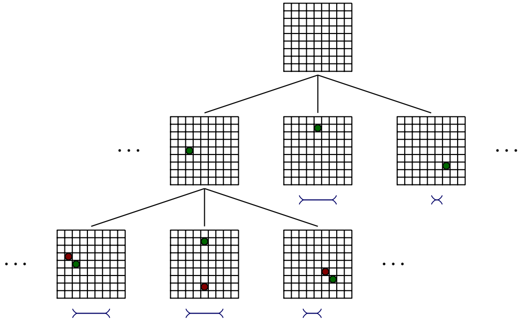

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Results of Hints

- **Breakthrough**: **beat if possible** ca. 70% improvement
- **Gomoku**: **play near your stone** ca. 80% improvement

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Solver Requirements

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Current Solver Architecture

- **FO assignments**: represented directly
- **MSO assignments**: semi-symbolically

$$(1 \in X \wedge 2 \in X \wedge 3 \notin X) \vee (1 \notin X)$$

- Operations on MSO assignments: **use SAT solver**, **CNF-DNF again**
- Are **BDDs** better? **Unclear**

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Learning formulas

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- Formula should **separate high-confidence** good from bad states
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